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SEMI-ANNUAL PROGRESS REPORT TO THE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

N.A.S.A. Research Grant NsG-544

Louis R. Bragg, Case Institute of Technology

May 1, 1964

1. Introduction. The research grant referred to in this report was awarded on November 1, 1963. The objective of the proposal behind this grant was to support studies in the combined areas of Special Functions and Solutions of Partial Differential Equations. At the present, three of the topics referred to in the proposal are under investigation and this research is at least partially supported by funds from the grant. Two of these topics are being investigated by two graduate students at Case Institute of Technology who wish to use the results for preparing Doctoral dissertations. I am serving as theses adviser for these students. At the moment, research for two of these topics is nearing completion while the research on the third topic is now beginning to take an apparently fruitful direction.

Specifically, the three topics being treated are:

(a) ~~The~~ Radial Heat Polynomials and Solution Representations ~~of~~ of Radial Heat Flow,

(b) Expansions of Solutions of Initial Value Problems (on finite intervals or bounded regions), and

(c) Polynomial Decompositions of Solutions of a Class of Initial Value Problems (in unbounded regions).

The remaining parts of this report will indicate the progress made on these topics, intended direction of further research, and expenditures made from the grant.

2. Research in Progress. In this section, the aims and status of research on each of the topics (a), (b), and (c) above will be discussed. A statement will be given pointing out how funds from the grant were used in connection with research or the reporting of research results relating to these topics.

(a) My studies connected with the Radial Heat Conduction problem are nearing completion. The problem being considered is that of obtaining [illegible] representations for solutions of the problem

$$(1)\quad U_t(r,t) = \Delta_n U(r,t), \quad U(r,0) = \phi(r).$$

where Δ_n is the Laplacian operator in radial coordinates in n-space. A

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formal integral representation for the solution of (1) has the form

$$(2) \quad U(r,t) = \int_0^\infty K(r, \xi; t)\, \phi(\xi) d\xi \ .$$

We define the radial heat polynomials by

$$R_k^n(r,t) = k!\ (4t)^k\ L_k^{\left(\frac{n}{2}-1\right)}\left(-r^2/4t\right)$$

where $L_k^{\frac{n}{2}-1}(X)$ is the generalized Laguerre polynomial of order k and index $\frac{n}{2} - 1$. The functions $\tilde{R}_k^n(r,t)$ associated with the $R_k^n(r,t)$ are defined by

$$\tilde{R}_k^n(r,t) = T_A\, R_k^n(r,t)$$

where T_A denotes the Appell transform. The set $\{R_k^n(r,t)\}$ is bi-orthogonal to the set $\{\tilde{R}_k^n(r,t)\}$ relative to an appropriate weight factor on $[0,\infty)$. We can decompose the kernel $K(r,\xi;t+s)$ in (2) into an infinite sum of products of radial heat polynomials and their associated functions. The problem remaining is to examine in what sense the solution (2) above can be expressed in terms of either the R_k^n or the $\tilde{R}_k^n$. The connection of these functions to the Laguerre polynomials is of real interest here. Results of this research will extend certain of the work of Rosenbloom and Widder [3] for heat flow in an infinite rod. The basic polynomials entering into the case they treat are the Hermite polynomials.

(b) This item is the thesis topic of one of my Ph.D. advisees, Mr. William J. Davis. Mr. Davis has received support from the grant in the following two ways:

(i) He has used travel funds to report his results to the American Math. Society (Notices of the American Math. Society, Abs. 611-59, April 1964) and

(ii) He has used much of my time for receiving directions and advice on his research. This time has been supported in part by the grant.

The topic Davis treats is concerned with representing solutions of a class of initial value problems considered on finite intervals in terms of solution transforms of a class of polynomials. Associated with the given partial differential equation is a formal solution operator which transforms polynomials into classical solutions of the equation. It is convenient to choose as a basic set of polynomials those generated by a generalized Appell generator (see [1]). By the use of bi-orthogonal sets of functions on the given interval (one set of which includes the given polynomials), the initial data can be represented in terms of the polynomials in the L_2 sense. The

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formal solution operator is applied termwise to the polynomials in the representation to obtain a formal series solution. The problem then is to determine in what sense the transformed series is a solution to the initial value problem, whether in the generalized or classical sense. The type of convergence is determined in terms of point-sets, called convergence sets. Applications of this representation theory are made to questions of stability and well-posedness.

Mr. Davis will use this for a Doctoral thesis. After completing the necessary examinations, he plans to submit this work for publication giving appropriate acknowledgement to the grant. You will be notified at that time and will be sent a copy of the submitted manuscript.

(c) Mr. Allen T. Hopper, a Ph.D. candidate in Mathematics at Case Institute of Technology, is now in the process of attempting to extend the work of Rosenbloom and Widder in a different direction from that in (a). Let $X = (X_1, \ldots, X_n)$ and consider the problem

$$(*) \quad U_t(X,t) = P(D)\, U(X,t),\ U(X,0) = \phi(X)$$

and $D = (D_1, \ldots, D_n)$, a differential operator. Numerous of these problems have the integral solution representation

$$(**) \quad U(X,t) = \int_{E_n} \phi(\xi)\, K(X-\xi\ ,\ t)\, d\xi$$

with K a known kernel. From the results of Ladyzenskaya [2], growth bounds are known for certain of these kernels. By the use of generalised Hermite polynomials and associated bi-orthogonal functions, Hopper obtains a decomposition for K. He now must establish growth bounds on these polynomials and associated functions to permit the interchange of integration and summation in order to get the series representation for $U(X,t)$ from $(**)$.

Mr. Hopper is receiving his entire support from this grant.

3. Planned Research. At the present, no major deviations from the originally proposed research is being planned. Quite naturally, a number of smaller problems have arisen from this work which will require special attention such as growth properties of certain special functions, determinations of suitable approximations to non-linear initial value problems, and stability questions. These, of course, fit into the original proposal. However, a number of them may become quite involved. In addition to the topics now being treated, the following topics are most likely to receive attention for the remaining six months of this first year's grant.

(a) Detailed asymptotic structure of generalized Hermite polynomials and

(b) Investigations in non-linear problems of the initial value type.

4. Expenditures. A detailed accounting of funds spent on this grant is being sent to N.A.S.A. by the business office of Case Institute of Technology.

REFERENCES

1. R. P. Boas and R. C. Buck, Polynomial Expansions of Analytic Functions, Springer-Verlag, Ergeb. der Math. und Ihrer Grenzgebiete, Berlin, 1958.

2. O. A. Ladyzenskaya, On the Uniqueness of Solutions of the Cauchy Problem for Linear Parabolic Equations, Math. Sbornik, Vol. 27, 1950, pp. 175-184.

3. P. C. Rosenbloom and D. V. Widder, Expansions in Terms of Heat Polynomials and Associated Functions, Trans. Amer. Math. Soc., Vol. 92, 1959, pp. 220-266.